Promoting Argumentation: Focus on Tasks

Module 2: Promoting Argumentation: Focus on Tasks

This module is the second of five modules created for professional learning purposes as part of the Bridging Math Practices project. An Overview for our facilitation guides and the modules is available at http://bridges.education.uconn.edu/argumentation-pd-modules/. This module can be used independently or in conjunction with one or more of the other four modules. We encourage users to become familiar with the set of materials and then adapt them to your particular needs and timeframe.

This Facilitation Guide includes the following:

- Goals for Module 2
- Background Information on tasks and
- List of Materials Needed for Module 2
- Timing Table for Module 3 Activities
- Implementation Guide and Possibilities
    - Detailed description of each activity and suggestions for implementation
- References
- Additional Resources

All handouts and other materials for Module 3 can be found at http://bridges.education.uconn.edu/norms-and-routines/

Goals: Module 2

Participants will

- Develop a deeper appreciation of argumentation and its potential in the math classroom
- **Analyze and evaluate tasks** to determine how they support argumentation in the math classroom
- **Identify and modify argumentation tasks to** prompt argumentation for a variety of instructional purposes by using three conceptual lenses

Overarching Questions for 5-Module Sequence

- What is a mathematical argument? What "counts" as an argument?
- What is the purpose(s) of argumentation in mathematics? In the math classroom?
- How do we organize our classroom to support student participation in the practice of mathematical argumentation, and to support them in developing their proficiency with argumentation (both verbal/interactive and written forms)?
- What does student argumentation look like at different levels of proficiency?

Background Information:

The focus of this module is on tasks, and how tasks can support (or impede) student participation in argumentation during mathematics class. The activities attend to analyzing and modifying tasks in order to develop ways of thinking about how tasks support argumentation and how they can be strengthened in that respect. It is important to note that tasks *set the stage* for the work the class will do. Tasks alone are certainly not enough. However, without a strong task that provides some opportunities for argumentation, it is extremely unlikely that students will engage that practice.

If you are interested, Henningsen & Stein (1997, 2002) report on an information study about tasks and their implementation. They found that tasks are about the potential – they are necessary, but insufficient, conditions for high quality mathematical work. We have included references to brief articles about this study in the Additional Resources section.

It may be important to emphasize during this module that a task may not require *written* argumentation, but the task may still support student argumentation. Particularly as students tackle new materials, or for student for whom recording ideas in written form is a challenge, one may not see a prompt that asks for a written argument. As participants examine the *potential* for the task to support argumentation, it may be worthwhile to help them think through how to generate those conversations and have students share ideas. These implementation strategies are a major focus of Modules 3 & 4.

Materials:

Handouts
PowerPoint slides (draft slides provided)
Projection capabilities
Access to internet and ability to play audio/video (or have TED Talk video downloaded in advance to play off a computer)
Participant tasks – 3 copies ***NOTE:*** The Bridging-to-Practice work for the Workshop model requires participants to have brought a task with them to the meeting for Module 2.

Workflow Table for Module 2

Module activity and focus	Estimated Timing		Materials
	Monthly (1.5 hrs)	Workshop (3.5 hrs)	
<u>Opening Activities</u>: PLC format: Participants share their "Bridging to Practice" work Workshop format: Community Building and/or Problem Solving	10 mins	(as appropriate for workshop timing)	Handout: Opening Activities Template
<u>Activity 2.1 Brainstorm Activity</u> Participants are asked to brainstorm "What is the value of argumentation in the classroom?"	10 mins	10 mins	Handout 1: Value of Argumentation in the Classroom Brainstorm Page
<u>Activity 2.2 The Role of Tasks and Introducing Three Lenses</u> Facilitators share an overview of the conceptual lenses that will be used, as well as an introductory TED Talk focused on how tasks offer different opportunities for students to think.	10 mins	10 mins	Handout 2: Overview of Lenses for Analyzing Argumentation Tasks
<u>Activity 2.2 - Lens 1: Engagement</u> Participants analyze and modify sample tasks to make them more likely to prompt argumentation	25 mins	40 mins	Handout 3e: Lens 1 Elementary; Handout 3s: Lens 1 Secondary
<u>Activity 2.2 - Lens 2: Student Learning</u> Participants analyze sample tasks to identify the different learning goals a teacher might pursue when using tasks that involve mathematical argumentation	25 mins	40 mins	Handout 4: Lens 2: Purposes for Using Argumentation Tasks to Support Student Learning; Handout 5: Lens 2: Purposes of Argumentation Tasks
<u>Activity 2.2 - Lens 3: Teacher Purpose</u> Participants consider the different ways argumentation tasks can be used strategically to inform instruction	0 mins	30 mins	Handout 6: Lens 3: Teacher Purpose – Informing Instruction
<u>Activity 2.3 Bridging to Practice</u> Monthly PLC Format: Explain work be completed between modules Workshop Format: Task development, vetting, and modification	5 min	70-75 mins	Handout 7: Bridging to Practice: Task Analysis; Handout 8: Guided Discussion of Tasks for Argumentation

<u>Activity 2.4 Module Closure</u> Participants reflect on experiences	5 mins	5-10 mins	

Implementation Guide and Possibilities: Module 2

Opening Activity

Monthly PLC Format

In the monthly PLC, you might organize participants into pairs or groups of three to debrief their Bridging-to-Practice work from Module 1. You may have asked participants to bring with them a task that they have used, or would like to modify to use, to support argumentation.

Another option is to revisit the Community Agreements. In general, we recommend that you provide a copy of the Community Agreements that participants generated at the beginning of this module, or sometime soon thereafter (e.g., end of this module). We also suggest that you offer an opportunity to extend, revise, or make suggestion about particular agreements that participants might wish to give special attention to during the upcoming work. For the group to continue to work well together, and have the opportunity to address any issues that come up, such work will be important. We do not recommend revisiting the Community Agreements each time you meet, but periodic revisiting is likely useful. You do not want to use these in a reactive way, only when things seem to have gone awry. Instead, using them in proactive manner can help keep space open for discussions and allow issues to be addressed, as needed.

Workshop Format

In the Workshop Format, you might pursue one of the suggestions above. In addition, you might have participants do a math task, to provide an opportunity to engage argumentation. You might even consider giving two or three different versions of a task to create an opportunity to analyze the mathematical work prompted by the different task structures, or perhaps even, which versions did and did not seem to offer support for argumentation.

Module Objectives

The module objectives should be introduced either prior to Activity 2.1: Brainstorm Purposes of Argumentation in the Classroom or after.

Participants will

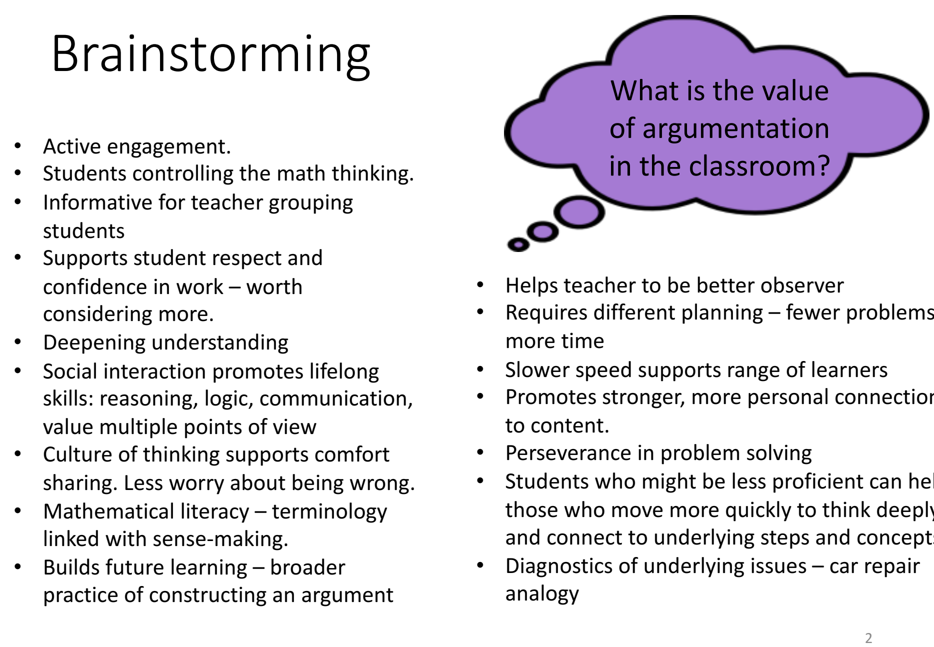
- Develop a deeper appreciation of argumentation and its potential in the math classroom
- **Analyze and evaluate tasks** to determine how they support argumentation in the math classroom
- **Identify and modify argumentation tasks to** prompt argumentation for a variety of instructional purposes by using three conceptual lenses

Activity 2.1: Brainstorm Purposes of Argumentation in the Classroom

The purpose of this activity is to provide participants with the opportunity to reflect on and extend their thinking about how argumentation is already used in their classrooms and other ways it can serve as a valuable practice and resource for learning. This activity is one that could have also been done during Module 1 (with some time adjustments). From the brainstorm and subsequent discussion, we hope that participants realize how powerful argumentation is as a practice and how it can contribute to many valued goals they already hold and try to promote. In addition, however, it is an important goal in and of itself to help students get better at the practice of mathematical argumentation.

Provide participants with 2-3 minutes to jot down their own brainstorm about the value of this practice. This can be followed by a partner share or discussion in small group where participants articulate their ideas and perhaps extend their list. This step can be skipped in the interest of time. The whole group then generates a list. A blank slide has been provided in the draft slides as a place to record participant ideas. These ideas can also be recorded on a white board, chart paper, sentence strips, googledoc, etc. (The value of a medium such as Google documents is that all participants can contribute at the same time and build on or link to one another's ideas.)

Participants may generate quite a long and varied list. Here is one list generated by the participants in one of our enactments.



You may also wish to press on points where they see potential conflation of ideas. For example, in the above list, one contribution is "perseverance in problem solving." How argumentation supports this type of perseverance is not obvious, and so the facilitator can follow up and ask the contributor to explain the connection s/he sees.

In thinking about the larger themes, we offer the following set of valued goals that argumentation can support – a list which is grounded in our experiences with teachers and some of the research literature.

- Supports the development of **conceptual understanding**
    - including helping to raise up and address/sort out **misconceptions**
- Develops students' **vocabulary and language skills** (verbal and written)
    - May be particularly important for English learners
- Develops **logic and reasoning** skills – especially deductive logic
- Promotes **collaboration** skills
- Helps students see themselves as **mathematical thinkers** and capable of deciding if something is right/wrong or true/false; math reasoning becomes the authority (and not what the teacher or the book says)
    - Shifts students' views of what math is (not a bunch of procedures to be memorized – it's about ideas and reasoning)
    - Can help manage status issues (or disrupt social status hierarchies) – it's not about who is fast, or usually right – it's about the line of reasoning offered.
- Supports **formative assessment;** provides the teacher with valuable **information about students' understanding** and ways of thinking
    - May be particularly valuable for students with uneven prior knowledge
- Makes math more **engaging** and **fun**
- Provides students with ***access*** to the ideas and how people reasoned (math looks less like magic; it pulls back the curtain)

In wrapping up the conversation, we suggest that you highlight some ideas. The following are three possible highlights from this activity:

1) Connect some of the comments to the day's theme of tasks and the three lenses (i.e., promotes student learning of content, as well as argumentation; provides teachers with valuable information)
2) Emphasize how argumentation is valued for a wide range of reasons, and likely contributes to teachers' extant goals for their students
3) Highlight the idea that it may be uncommon to use mathematical argumentation solely for the purpose of helping students get better at argumentation or writing mathematical arguments. While these goals are valued, argumentation is such a powerful learning practice that it can be used regularly to reach other ends.
    a. Alternately, if participants are instructors of proof-based courses such as geometry, and see proof as an end goal of their course, it may be important to point out that argumentation is an everyday event and those skills need to be fostered in less formal contexts extensively if students are to ultimately be successful with the particular form of argumentation called proof.

Activity 2.2 The Role of Task and Using Three Lenses

Overview of activity:

In this multi-part activity, participants are introduced to three "conceptual lenses" that can be used to help them think about tasks in the math classroom, specifically those that support mathematical argumentation. An important key take away here is that tasks are critical for setting up possibilities but (a) alone, tasks are not enough – how they are implemented matters and (b) no one prompt or line in a task makes it an "argumentation task."

A video from Dan Meyer can be used to set the stage. TEDxNYED – March, 2010
https://www.ted.com/talks/dan_meyer_math_curriculum_makeover?language=en
We suggest showing the first 6:36 minutes of this video, though you might opt to not show it, or to show all of it. It is a well-known video, so some participants may have seen it before. The main purpose of showing this video is to focus in on tasks and how they engage (or not) students. Meyer is clear about how the over structuring of tasks leads to lack of engagement and lack of reasoning. He promotes the idea that more open beginnings allow students to start thinking and reasoning, and then, as the discussion unfolds, structure can be added as needed. These ideas blend task design with implementation.

The next part of the activity introduces three "conceptual lenses" that participants will be asked to use with tasks to support discussions about whether and how the tasks support argumentation, and what the different valued outcomes (learning goals) might be. This can be a good place to connect back to the discussions from the first 2.1 Brainstorm Activity.

Here we provide an overview of the conceptual lenses:

	Focus
Lens 1 Engagement in Argumentation	Focus on whether or not the task, as written, is likely to prompt students to engage in mathematical argumentation. If not, how might one modify the task for this purpose
Lens 2 Learning Goals	Focus on learning goals one might pursue when selecting an argumentation task. Some purposes for using an argumentation task include: to promote conceptual understanding; target misconceptions; make sense of how two representations show the same idea, or which is more useful in a particular case; expose students to multiple approaches in problem solving and show connections among topics.
Lens 3 Informing Instruction	Focus on what a teacher might gain for purposes of adjusting instruction or better knowing his/her students and where they are with their thinking. Argumentation can elicit prior knowledge, reveal students' strengths or preferred way of reasoning about an idea, or the depth of their knowledge about a concept.

Note: There is a lot to think about across these three lenses. You may choose to highlight one or two of these three lenses, instead of all three. (Perhaps introducing the other(s), but working with less extensively.)

Lens 1 is a top priority as selecting and modifying tasks is a necessary skill for anyone who will be supporting student argumentation. Working with Lens 1 also provides the opportunity for the group of participants to continue developing its collective understanding of mathematical argumentation, as discussing these tasks will bring out these nature of student participation and whether it is argumentation.

Lens 2 is valuable for understanding how argumentation can address multiple goals and how sometimes a teacher is using argumentation primarily in service of other valued learning goals (e.g., conceptual understanding), and at other times, the focus may be on helping students get specifically better at argumentation itself.

Lens 3 is a valuable reminder that teaching involves a feedback loop and teachers need to constantly be learning about what their students know. It is also a message that argumentation tasks can/should be implemented at any point in time or during the course of a unit, and not only "after they learned X." Sometimes teachers think they must teach content before students can be asked to do things like provide arguments. Argumentation however is useful for assessing prior knowledge, developing ideas, providing opportunities for sense making, and as an assessment/information tool once student mastery of a concept is expected.

Introduction

A draft PowerPoint slide provides an overview of the lenses, as well as a handout. One option is to introduce the lenses by connecting the ideas back to the Brainstorm Activity. It may be helping in the introduction to emphasize that this is *one way* to think about tasks. There are many other lenses that someone could bring to analyzing tasks. We offer these since we have found them helpful in thinking tasks in relation to argumentation, but participants may have other approaches.

Activity 2.2 - Lens 1

With Lens 1 (focus on engagement in argumentation), participants are asked to analyze a task and decide whether the task is likely to engage students in argumentation. The general structure of this activity (with suggested times) is: discuss examples together (5-10 mins); small group work (20 mins); full group debrief or jigsaw debrief (10-15 mins), for a total of 40 minutes.

Three slides, with three sample prompts, are included in the draft slides. You can ask participants first if they expect the Original question to engage students in argumentation. You can ask them to generate ideas for the revision, or share the revision and ask them to discuss.

Note that when answering the question, does the task engage students in argumentation, participants often answer the question "Well, it could" or "it doesn't *have* to" as they explore all the different ways student might respond. While true that any one prompt might or might not prompt argumentation, as students can do a wide range of things in response to a task, the focus should be on what the task requirements are, as written – what must a student offer to respond to the prompt adequately? The following question can offer guidance:

> If a student does what is asked of him or her by the task (with reasonable interpretation), will s/he be likely to engage in *constructing viable arguments and/or critiquing the reasoning of others*?

For example, in response to the first prompt, "What fraction of the rectangle below is shaded?" a very reasonable response is "¼" with no additional explanation provided. Yes, a child might offer more, but the task doesn't require more to complete task.

For the revision (that asks the student to agree or disagree with Laura's claim), a reasonable response to the prompt would begin "I think Laura is correct because…". To do the task, a student needs to offer a reason (argument). Note that what follows that "because" will determine whether the child has offered a mathematical argument or not. A child could say "I think she's not correct because she's wrong." There is not really much of an argument there. Nevertheless, the task itself puts students in a position of needing to offer an argument in order to reasonably complete the task.[1]

The table includes the issues referenced in the above discussion as well as other ideas that might come out in discussion.

Potential points of discussion for three examples on draft slides for Lens 1: Engagement

Issue/Participant Comment	Response/Commentary
In considering whether a task prompts argumentation, participants might comment: A student doesn't *have* to give an argument -or- A student *might* give an argument (even when the prompt is "What's 3 + 5?")	It is true that for any one prompt, a student could respond just about anything. This activity asks what the tasks requires of students to respond reasonably. To complete the task, would a student generate a mathematical argument? The view to take here is – If a student does what is asked of him or her by the task (with reasonable interpretation), will s/he be likely to engage in *constructing viable arguments and/or critiquing the reasoning of others*?
Are there key questions or key words that tell you it's an argumentation prompt? This question might be offered after seeing the "explain why or why not" for the first prompt and the addition of "write a mathematical argument to support your decision" in the second prompt.	No one sentence or one key word does the trick. A sentence like "write a mathematical argument to show…" is pretty clear, but for the most part, our language doesn't help us enough. Prompts like "Explain why or why not" or "how do you know" might elicit arguments from students, but could also elicit students showing their work clearly. The important idea here is that teachers must make meaning of these questions *with students.* What it means to "explain why?" is something student will learn from being in math class with a particular teacher. Similarly, what it means to "show how you know" or "explain how you know" will depend on the expectations students have internalized for such prompts.
For the prompt, The Race - participants think the original *does* prompt argumentation because it involves writing. Others may think this because you have to interpret the graph and really	The original prompt is not one we consider likely to prompt argumentation. A child can write a story without offering any supporting arguments for why the parts of the story must be true based on the graph. For example, a student could write – Juan was behind at the beginning, but he caught up with Antonio around minute 7, and then he finished first! The

[1] It is not uncommon for students to offer a response something like, "She's right because I did the problem and got the same answer" and the student might even show his or her work. Though not a strong argument, the student essentially is arguing that s/he did an independent verification and got the same answer, so Laura must be right. The question remains then: how does the student know that s/he is right? The student's answer then would require an argument to support it.

understand it in order to write a story about it.	child's reasoning for this, however, is not available to use. One might assert that a child has to do some kind of justification in her head perhaps about the claims she makes about the race. For example, if she decides to announce that Antonio was in the lead for the first several minutes, she has to have read the graph a decided that was true based on the evidence offered and his/her knowledge of graphs. We could make this kind of argument about just about any prompt, as a child has to answer based on something. Not surprisingly then an argumentation task requires more than just the *possibility* of thinking.
What's the difference between a task that prompts students to *write an argument* versus a task that prompts students to *engage in argumentation?*	There is indeed a difference between a task that prompts for students to produce a verbal or written *argument* and a task that prompts students to engage in mathematical *argumentation*. These two types of tasks may have rather different prompts – from "share your thinking with your partner" to "write a mathematical argument to show your chain of reasoning for how you know your result is true." Both can support mathematical argumentation. In the latter case, it's more clear where that comes in. In the former, the teacher will need to be sure to follow up so that initial sharings from students that look more like saying one's steps, or are underdeveloped thoughts, can be drawn out and focused on reasoning.
Participants might suggest other changes to make the task "better" that do not necessarily strengthen the attention to argumentation. For example, participants might suggest changing the tense or wording to reduce the language demands (and therefore more likely to offer an argument).	Acknowledge that such revisions make the task more accessible. In that sense, those revisions make the task more likely to prompt argumentation. In addition, however, make sure participants are attending to how to ensure the task – regardless of particular needs of the individual students – will promote argumentation. As a follow up, ask if the suggestion changes the nature of the mathematics students would do (with the prompt understood).

After sharing and discussing some examples (note: not all examples need to be discussed, or discussed in as much depth), participants work in small groups on the Lens 1 handouts where they modify prompts to further promote argumentation. There are two versions of the handout – one for elementary school teachers (Lens 1: Elementary) and the other for secondary teachers (Lens 1: Secondary). We suggest that participants work in groups of 3 and have about 20 minutes to complete the handout.

Two tables have been included below. Each (one for the Elementary version of the handout and one for the Secondary version of the handout) shows the original problem, some commentary, and a sample revised prompt.

After the small group work, the full group can then reconvene. Participants can share their revisions (e.g., write on the board; jigsaw with another group) as the basis of further discussion, raise questions about particular revisions, etc. Facilitators might also ask for revisions with particular qualities, for example, a revision that directly prompts for students to write out an argument, or a revision that directly prompts discussion (but no written argument, at least not initially). In the draft PowerPoint we have included a slide for each elementary prompt and for each secondary prompt that you can use, as is helpful, to display prompts during discussion.

In the table we have offered some brief comments and potential revisions. Please note there are many, many more possible revisions, several of which are likely superior to those offered here. Do not limit yourself or participants with this table! This table is meant to offer some beginning ideas, but the conversation will likely move well beyond this.

Elementary Prompts on Handout 3e: Elementary - Potential revisions and comments

Original Prompt	Comments	Potential Revisions
Task E1: Fill in the missing value that makes the statement true. a) 10 + 5 = 2 + 3 + ____ b) 500 ÷___ = 10 c) 25 x 10 x 4 = _____	As is, this task would not require students to express reasoning, or perhaps not even reason. Students *might* reason, for example, the first blank must be 10 because 2+3 is 5, which accounts for the 5 on the left side. They do not however have to do this. They can use their computational skills fairly exclusively.	Task E1-Revised: Keep the problems; change he instructions: Without using a pencil, figure out the value that makes the statement true. You have 5 minutes. Then you will share your answer with your partner and explain your reasoning and how you know. *By changing the task to a mental task, students are more likely to reason about the values. If students can do the computations in their heads, perhaps different numbers should be chosen.*
Task E2: Alexa is training to bike 70 miles. During her first week of training she bikes 12 miles. During her second week she bikes 24 miles, and by her third week she bikes 36 miles. On what week does she bike close to 70 miles?	As is, students report “Week 4” as their answer. This may be supported by some work, perhaps showing 12+12=24; 24+12 = 36; etc. The task, as is, does not require the student to offer an argument.	Task E2-Revised Alexa is training for a race. During her first week of training she bikes 12 miles. During her second week she bikes 24 miles, and by her third week she bikes 36 miles. When will she be ready for a 70-mile race? Explain which week and how you knew, and any assumptions you had to make.

Task E3: The coordinates of the vertices of figure ABCD are A(4, 3), B(8, 3), C(4, 6) and D(8, 6). Is figure ABCD a rectangle?	A strength of this problem is that students can tackle it many different ways, and have to make a claim about whether the figure is a rectangle or not. It all but begs for evidence and an argument to support the claim. In some classrooms, with strong norms around argumentation, this prompt may be sufficient. In other classrooms, students might not offer more than an answer (claim).	Task E3-Revised Keep the prompt and add: "Give a mathematical argument to convince a friend in Ms. X's math class that this is or is not a rectangle." *Another revision might change the nature of the mathematical work a student does, but similarly taps into the definition and properties of a rectangle:* The coordinates of the vertices of figure ABCD are A(4, 3), B(8, 3), C(4, 6) and D(?, ?). (i) Find coordinates for vertex D to make ABCD a rectangle. (ii) Is this the only vertex that works (to make it a rectangle). Explain why or why not, and how you know.

Secondary Prompts on Handout 3s: Lens 1: Secondary – Potential revisions and comments

Original Prompt	**Comments**	**Potential Revisions**
Task 1: Solve each of the following: a) 3x + 5 = 2x – 6 b) 4x + 3 = 4x – 5 c) 2x– 10 = 2x– 10	This task likely will elicit procedural work to "solve for x" and then interpreting the result (e.g., what does it mean if solving yields x=x or 0 = 0?). Argumentation is not required to respond to the prompt. That said, a student could look at, for example, (b) and report that there is no solution as there is no way for 4 times a number to have the same value when you add three to it and when you subtract 5 from it. Alternately, the student might report that the two expressions in (b) are both lines, and these lines are parallel because the two lines have the same slope, and therefore never meet, which means they are never equal for any given x (so no solutions). The revision suggested aims to prompt more reasoning like this (which is unlikely in the original).	Task 1S-Revision Directions can be modified to read: <u>DO NOT</u> solve. Discuss with a partner the solution(s) to each equation. Support your thoughts with a mathematical argument. *Try to come up with more than one argument if you can.*
Task 2: Alexa is training to bike 100 miles. During her first week of training she	This prompt does not elicit a mathematical argument as written. Students write an equation and then	Task 2S-Revision *Same set up. Revise question to:* If she continues to advance in

bikes 12 miles. On her fifth week she bikes 40 miles. Write an equation to represent her training progress and use it to determine on what week she will be able to bike 100 miles.	apply that in context.	this same pattern, on what week will she be able to bike 100 miles? Write a mathematical argument to support your answer. *Another revision might be the following:* *Same set up* Write an equation and justify your equation. That is, explain how you know your equation appropriately models Alexa's training progress over time (weeks).
Task 3: The coordinates of the vertices of parallelogram ABCD are A(-4, -3), B(5, 6), C(8, 3) and D(-1, -6). Determine the slopes and lengths of the sides to verify that it is a rectangle.	This task is in the ballpark for argumentation, asking students to verify the parallelogram is a rectangle – a prompt which should lead them to think about the definition of a rectangle and how it is a parallelogram with special properties. That said, the prompt could be strengthened. We suspect most students will not realize the mathematical work they are doing as the prompt is directive in telling students *how* to approach the mathematics, and in some ways is misleading. Note also that verifying that this parallelogram is a rectangle does not actually require computing lengths of sides, as the opposite sides of a parallelogram are equal in length. A student really only needs to calculate the slopes.	Task 3S-Revision *Keep the same set up. Change the prompt to:* After some calculations, Jasmine has classified this as a rectangle. Use your knowledge of parallelograms and rectangles to create a mathematical argument that will verify her conclusion. *Note this prompt opens up space for students to approach the argument however they wish. It also asks them to use what they know about the two types of figures. This is also a directive in some respective, but its directive about the foundation of the argument and not the specific methods/computations, which we think is beneficial, and which is in contrast with the original.*

2.2 Lens 2: Student Learning Purpose – goals teachers might pursue when implementing argumentation tasks

Lens 2 focuses on the different learning goals that a teacher might pursue when engaging students in an argumentation task. These ideas came out of a question from some of our Bridges participants after they have been working with argumentation for a few months. They wondered what are the different "kinds" of argumentation tasks that one could pose. They were interested in this both to help them think about whether they were using argumentation fully, as well as wondering about different types of assessment prompts that could be used under the broader umbrella of argumentation.

From this question and an examination of tasks, we develop these four categories. Note that the categories are not distinct, and the set may not be fully comprehensive. These are, however, four prominent types of mathematical argumentation tasks that can be used, each of which serves different learning goals.

A. **Goal: Students produce better arguments**
B. **Goal: Students develop conceptual understanding**
C. **Goal: Students mathematize problem situations and interpret meanings of solutions in context**
D. **Goal: Students make sense of and compare across multiple approaches and multiple representations**

The Timing table allocates 40 minutes to this section of the module. One breakdown for this time, as done above, is 10 minutes for guided discussion about the learning goals/purposes and examples; 20 minutes of small group work, as participants apply the lens in relation to tasks; and 10 minutes of full group debrief, to help work through questions that arose and emphasize key points.
Note that the structure of this activity does not provide space for participants to *create their own task(s)* in relation to one or more of these goals, nor to modify these tasks for other purposes. Such activities are valuable and can be worked in at the facilitator's discretion and with appropriate time. Alternatively, they might prove interesting options for Bridging to Practice work, depending on the interests of your group.

See Handout 4: Lens 2: Purposes for Using Argumentation Tasks to Support Student Learning for an elaborated description of the type of mathematical activity that we see as connected to each Goal. The draft slides include an example of a task that showcases each goal. Be sure to use this set of goals and the slides as a resource to help participants make sense of differences and see the power of argumentation. It is important to avoid presenting this as a framework to label tasks. The actual labels are less important than developing the idea that there is not just one kind of argumentation task, and that there are many different learning goals that argumentation can help a teacher pursue.

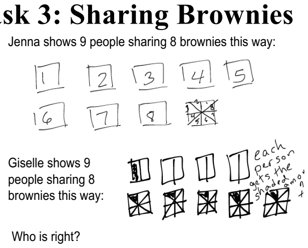
After making sense of the different purposes – and you can open space to see if participants would like to identify additional purposes – provide participants with Handout 5: Lens 2: Purposes of Argumentation Tasks.

In the table below, we offer some comments here about the purpose(s) of the task and some points or questions that might be raised in relation to each task. As with the Lens 1 work, we have included the tasks on the PowerPoint slides in case it is useful for discussion. With time, you can encourage participants to experiment with revising prompt to address other purposes. (The goal here is to show how changes in wording may keep argumentation as part of the task, but shift the focus of the mathematical work.)

Goal: Students produce better arguments
Goal: Students develop conceptual understanding

Goal: Students mathematize problem situations and interpret meanings of solutions in context

Goal: Students make sense of and compare across multiple approaches and multiple representations

Task	**Purposes**	**Other comments**
Task 1: Jasmine & the Square The coordinates of the four vertices of figure ABCD are A(4, 3), B(8, 3), C(4, 6) and D(8, 6). Based on the differences between the coordinate points, Jasmine believes figure ABCD is a square. Do you agree with her? Write a mathematical argument to support your answer.	This task targets conceptual understanding of a square. Students have to apply the definition of a square to this situation and determine (a) if all sides are of equal length and (b) if the sides meet at right angles in order to agree or disagree with Jasmine.	Students are also asked to write a mathematical argument, which has the potential to help them learn how to produce better arguments. This outcome would likely come about through discussion or feedback from peers or the teacher. Similarly, if the teacher had a discussion about different approaches, this could contribute to the last coal.
Task 2: Bike Training Alexa is training to bike 70 miles. During her first week of training she bikes 12 miles. During her second week she bikes 24 miles, and by her third week she bikes 36 miles. If Alexa continues with the same biking pattern each week, when will she be able to bike 70 miles? Write a mathematical argument to support your answer.	This task targets mathematizing the problem situation. Students might create a graph, equation, table or other representation of the situation to determine the point at which Alexa will be able to bike 70 miles. By asking for a written argument, students would need to connect their mathematical work to their asserted week.	As with Task 1, by writing an argument, students may have the opportunity to get better at argumentation. This will depend on feedback. Similar to above, if the teacher chooses to discuss different approaches – of which there could be many – this task could contribute to the goal of comparing across multiple approaches and representations. Note that Tasks 1 & 2 have identify "prompts" at the end, but students do different intellectual work.
Task 3: Sharing Brownies 	This task targets making sense of multiple representations and different approaches, as students must examine each representation of sharing and determine whether this was a reasonable way to mathematize the problem situation (which contributes to the third goal).	Depending on the implementation, the teacher can extend this readily into asking students what answer Jenna and Giselle each would get and the argument each would offer to support that assertion. (Note that most of the "argument" is embedded in the representation once we make sense of each representation. Students can be encouraged to notice that these representations require some explanation to help the reader make sense of the student's mathematics.)
Task 4: Kay is squaring numbers. She notices that when she squares a number, the result is *larger* than the original number. Here are some of her examples: $3^2 = 9$, $10^2 = 100$, $(-4)^2 = 16$	This task targets getting better at argumentation by asking students to consider Kay's conjecture and evidence. In part a, students find a supporting example. In part b, students reflect on	Also in support of helping students get better at argumentation, teachers might have a conversation about how Kay's three examples, plus the additional one generated by the student, is not enough to show that

She conjectures "the square of a number is always larger than the number." (a) Find another example that supports Kay's conjecture. (b) Is this conjecture always true (for all numbers)? If so, explain how you know. If not, revise Kay's conjecture so that it is a true statement.	the claim and examine its validity and/or assumptions involved. Three possible revisions include: *True for all numbers larger than 1.* *True for all numbers with an absolute value larger than 1.* *Kay has assumed she's looking at integers, and it's true for all integers except 0 and 1 as written. She can revise and say "The square of an integer is always greater than or equal to the integer."*	something is true for ALL numbers (or integers). This helps with the overall goal of helping students realize that inductive patterns are not enough in mathematics to show something is generally true. The task can also support developing number sense, helping students think about multiplication and potentially how multiplication makes a number larger or smaller (e.g., multiplying by a value between 0 and 1 results in a number that is only part of the original whole).

2.2 Lens 3 –Informing Instruction

This third lens emphasizes the idea that teachers are always learning from their students engaging students in argumentation provides a robust opportunity to learn more about students- thinking, whether prior knowledge, ways of problem solving, and depth of conceptual understanding, or misconceptions.

Questions teachers might ask themselves include:
<u>When</u> would I use this task? <u>Why</u> am I using it? <u>What</u> will I learn from it?

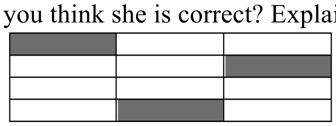
As we think about this third lens, informing instruction, we pose the following guiding question and subquestions:

What do you plan to <u>learn about your students</u> by using the mathematical argumentation task?

- Does the task help you learn about students' prior knowledge about a topic?
- Does the task help you learn about students' ability to apply or connect their knowledge of one or more topics from a lesson, unit or course?
- Does the task help you learn about students' degree of mastery of targeted skills or concepts?
- Does the task help you learn about students' ability to communicate their reasoning effectively and to make clear connections among their claims, warrants and evidence?

For the Handout 6: Lens 3 Informing Instruction, participants analyze four tasks – all prompts they have seen as part of Lens 1 or Lens 2. Note that we do not differentiate between elementary and secondary prompts, but you may wish to focus more attention on task prompts that your group may teacher. Alternately, you might choose to replace some of these task prompts with ones you know your participants teach, or that are at a grade or two above or below.

Handout 6: Lens 3 Informing Instruction task prompts and commentary.

Task	**What might you learn**
Please note that ALL 4 task prompts could serve to inform instruction in relation to all goals noted above – to tap into students' prior knowledge (or assess prior knowledge), connect their knowledge of one or more topics (which we point out below), gain information about the degree of mastery, or learn about students' proficiency in communicating their reasoning.	
Task 1: Jasmine & the Square The coordinates of the four vertices of figure ABCD are A(4, 3), B(8, 3), C(4, 6) and D(8, 6). Based on the differences between the coordinate points, Jasmine believes figure ABCD is a square. Do you agree with her? Write a mathematical argument to support your answer.	This task could provide information about students' understanding of a square, how they are working with definitions, and/or asking students to apply their knowledge of a coordinate plane *and* knowledge of geometric figures.
Task 2: Bike Training Alexa is training to bike 70 miles. During her first week of training she bikes 12 miles. During her second week she bikes 24 miles, and by her third week she bikes 36 miles. If Alexa continues with the same biking pattern each week, when will she be able to bike 70 miles? Write a mathematical argument to support your answer.	This task could provide information about students' understanding of proportional relationships or linear relationships (depending on how they approach the problem), and their preferred approach to this type of problem (i.e., what representation or tools they use to solve it). Note that this prompt is quite open, so it would not be ideal for assessing particular *skills* such as whether a student could construct an equation, graph the situation, or identify the slope and what it means. The openness however is beneficial in seeing how students put together arguments and how they approach and make sense of such problems.
Task 3: Is It ¼? Laura says that ¼ of the rectangle is shaded. Do you think she is correct? Explain why or why not. 	This task could provide information about students' understanding of fractions and equivalent fractions. It could be used as the idea of equivalent fractions is introduced, or to assess students' knowledge of this topic, as the prompt requires them to explain how that figure that seems to show 3/12 is also 1/4.

Task 4: Solving Equations DO NOT solve. Discuss solutions to each equation. Support your ideas with a mathematical argument. a) $3x + 5 = 2x - 6$ b) $4x + 3 = 4x - 5$ c) $2x - 10 = 2x - 10$	This task could provide information about students' understanding of what it means to *solve* an equation as well as their knowledge of lines, and how the graph of a pair of lines relates to the solutions of a related equation. This brings in students' think about slopes, and parallel, intersecting, and overlapping lines. Given this focus, the task can also be used productively at the point in a unit where the instructor hopes to prompt students to think about the relationship between the graphical representation of both expressions (sides) of an equation and the solution to that equation. As it would be challenging for the instructor to hear all the discussions among students, it wouldn't provide a systematic look at all students' reasoning and ability to communicate an argument, but the instructor would receive some information about that.

In the interest of balancing depth and breadth in the module overall, we have suggested that this is optional for the Monthly PLC format. These ideas can be woven in to other discussions for the monthly format, or addressed in a briefer discussion.

Summary

In your wrap up of the targeted work on the conceptual lenses, you can remind participants of the three lenses and ask them to reflect on the role of tasks in supporting mathematical argumentation. We hope that a big take away has been that ***tasks create possibilities, but tasks alone are not enough.*** In the draft slides, we have included a blank slide with this title, as well as a "hidden slide" with some points that may be raised by participants, or that you may find useful to raise to solidify some of the information in this module. Some of these may not be relevant or strike the right chords with your participants, so we encourage you to modify as appropriate. A potential prompt for participants is the following:

> *How has the work with these lenses helped you think about selecting, modifying and/or using argumentation tasks in your classroom? What questions remain for you? What would you like to learn more about?*

The last bullet we have included in the hidden slide on the "Task Create Possibilities" slide may require some additional commentary. "Secure content" is content often from a prior grade level, or content that the instructor can expect is well understood by his or her students. The idea is that sometimes it is challenging to determine a student's proficiency with argumentation when it's unclear if the difficulties lie with the content, communication of the ideas, or understanding what an argument is. When using "secure content" in a prompt, this can eliminate (or reduce) challenges related to a lack of understanding of the ideas. The Smarter Balanced Assessment Consortium uses this idea in creating performance tasks, where the content can be "below grade level" (secure content) and the level of difficulty in relation to communicating reasoning of problem solving is on grade level.

We have found that participants (at some point – perhaps in module 2, and perhaps later) are interested in being given or knowing where to find good tasks that engage students in argumentation. There are some good websites that offer strong tasks, which you might opt to share. (See Additional Resources section for some examples.) Participants can also be strongly encouraged to continue to work on *modify* the tasks (as was done with lens 1 examples) to prompt engagement in argumentation. Participants need to develop that skill, as teachers might be required to use materials to use and they will need to see how to adjust them for their students' needs. This adapting/modifying approach may be particularly important for working argumentation in on an everyday basis. Many of the tasks found on websites in compilations are very good problem solving tasks and are not intended to address targeted skill development or are not tasks that can be done in a shorter period of time.

We have also found that secondary teachers primarily – though elementary teachers as well – sometimes have questions about how argumentation plays out in the work they see as procedural and students "just have to know." For example, participants wonder about how argumentation relates to "solving 2-step equations" or finding the inverse of a function. We include those in the Additional Resources section as well.

Activity 2.4 Bridging to Practice Activity

As stated previously, the Bridging to Practice activities are a staple of this professional development that support participants to link the concepts of the PD with their work in classrooms and schools.

Monthly PLC Format

One option for the Bridging-to-Practice activity is to have participants do some deliberate work in relation to task design and implementation to support argumentation. Here is one version of the activity that could be used:

> Bridging-to-Practice
> Look ahead to what you will teach in the next month.
> 1. Select a task that you can use (as is, or modified) to engage students in argumentation. If you modify, keep track of the modifications you made.
> 2. Analyze the task using the three lenses. Use a copy of the Lenses Organizer. [How is argumentation supported? What is the learning goal? What might I learn about my students?]
> 3. Enact the task with your students. Make a copy of the students' work.
>
> <u>Please bring to the next meeting:</u>
> Original task, modified task (if applicable), and student work samples

For the opening activity for Module 3, participants can then share their tasks (including modifications and intentions), some student work, reflection on how it went, and then ideas for future work. The discussion can also include participants sharing challenges they found in modifying or implementing tasks, and asking for suggestions and input from other participants.

Workshop Format

Overview: For the Workshop Bridging to Practice activity, participants share a potential task for use in their classroom and receive input from others on their task, and particular question about its use, through a protocol-guided discussion.

The activity begins with individual work as participants analyze (and potentially modify) an argumentation task that they have selected – most likely something they have taught or expect to teach. Participants have 10 minutes to analyze the tasks in relation to the three lenses, guided by Handout 7: Bridging to Practice – Task Analysis: Viewing Through the Lenses.

This individual work is followed by a protocol-guided discussion in groups of three (Handout 8: Bridging to Practice – Guided Discussion of Tasks for Argumentation). We recommend that with such small groups, participants be groups with similar grade-level assignments. (It is not optimal to have two elementary teachers discussing an Algebra II task, or two secondary teachers discussing a task targeting developing counting strategies. There is value, but it requires too much background information to be filled in, and given the short time, we do not recommend it.)

The protocol is an 18-minute protocol, allowing for a couple minutes of transition to accommodate the overall 70-minute block of time for the Bridging-to-Practice activity. If participants are not familiar with using protocols, or need additional support in implementing protocols, we recommend that you adjust times earlier in the session to accommodate some additional attention to these areas. Alternately, you might choose to model a protocol guided discussion, model certain steps of the protocol, or co-facilitate in some instances (if additional instructors or knowledgeable personnel are available). You can also modify the feedback structure more significantly if you like.

We have found there are two key aspects of implementation that are needed to make protocols work well. The first is a clear question posed by the presenting participant. Often participants have many concerns and thought, and this comes out in a question that is not clear or asks multiple questions at once. As the facilitator, you may wish to model questions, or you may wish to walk around during the individual work time and check in with participants.
The second is embracing the discomfort of not being able to talk when you are the presenter. It can be helpful to identify this step in advance for participants. There is a tendency when someone is talking about something you created to want to "explain" in relation to each comment – "I thought about that, but I didn't do it because…" or "I don't think that would work because my students don't know ….") This "silent time" forces the presenter to take what s/he can from the comments and really listen, not taking any of the valuable time away from the ideas of others in the group. There is time for additional discussion, where, if needed, the presenter can offer some additional thoughts to help the discussion even better target his/her question.

Encourage presenters to listen and absorb as much as they can, and not feel they need to respond.

Module 1 includes some additional references and resources related to protocols and protocol guided discussions.

Activity 2.5. Closure

The closure can be used to elicit feedback from participants both in order to (a) see what they are understanding from the material, and/or (b) get information about how the facilitation, module organization, etc., are working for participants.
In addition, you can use closure to have participants reflect on their learning and next steps, for example, they might consider one or more of the following questions:

- How comfortable are you now?
- What did you learn?
- What questions do you still have?
- What are your next steps for supporting argumentation in relation to tasks?

References

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28,* 534-549.

A more practice-focused version of this research article is available as a chapter. Henningsen, M., & Stein, M. K. (2002). Supporting students' high-level thinking, reasoning, and communication in mathematics. In J. Sowder & B. Schappelle (Eds.), *Lessons Learned from Research,* (pp 27 – 35). Reston, VA: National Council of Teachers of Mathematics.

Kazemi, E. (1998, March). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4*(7), 410-414.

Lens 2, Task B in slides
2nd prompt is from the first part of a sample performance task that is available on the web at several sites, and seems to have been developed early on by SmarterBalanced
One source: https://www.hawaiipublicschools.org/DOE%20Forms/Testing/MathGrades9-11SampleItems.pdf

Lens 2, Task C in slides
Task C Figure and original prompt from NCTM (2000) *Reasoning and Proof,* p 189.
National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Additional Resources: Module 2

Another excellent talk, *Beyond Relevance & Real World: Strong Strategies for Student Engagement,* by Dan Meyer regarding tasks was given at the NCTM Annual Conference April, 2016 in San Francisco. It is available online at https://vimeo.com/163821742 . You can also read a bit about this talk and related ideas at http://blog.mrmeyer.com/category/design/presentation/

Smith, Margaret Schwan, and Mary Kay Stein. "Selecting and Creating Mathematical Tasks: From Research to Practice." Mathematics Teaching in the Middle School 3 (February 1998): 344–50.

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The Module 2 Facilitation Guide prepared by Megan Staples and Jillian Cavanna based on 2014-2015 and 2016 implementations of the Bridging Math Practices Project. Last updated October, 2016.