Module 4:

Prompting and Supporting Argumentation: Focus on Implementation & Classroom Discourse

Module Goals:

- Developing deeper understanding of argumentation and its potential in the math classroom.
- Analyze mathematics classroom discourse interactions that can support students to engage in argumentation
- Reflect on current instructional strategies to consider how they will promote discourse and argumentation in the classroom

Small Routines to Support Argumentation

How Do You Know? ------

GIVEN: $\overline{WX} \cong \overline{YX}$, Z is the midpoint of \overline{WY} .

PROVE: $\triangle WXZ \cong \triangle YXZ$



Are the two triangles congruent? How do you know?

Cross out the function that does not belong. Create a mathematical argument to support your decision.

$$y = -8x(x + 1)$$

$$f(x) = 6x^{2} - 1 - (6x + 1)$$

$$y = x^{5} + 3x^{2} - 5$$

Would you rather? - - - - - -

Create a mathematical argument to support your decision.



- - - - - - - -

Additional Pedagogical Routines to Support Argumentation – Everyday

Pedagogical moves or tools you likely already know

- Turn and talk
- Think-Pair-Share
- Quick Write (pre-write; check for understanding)
- Exit slip
- Journal prompts

General Pedagogical Moves

Talk Moves (Chapin, O'Connor, & Anderson (2003)

- Revoicing "So what you're saying is that it's an odd number?"
- Asking students to restate someone else's reasoning "Can you repeat what he just said in your own words?"
- Asking students to apply their own reasoning to someone else's reasoning "Do you agree or disagree and why?"
- Prompting students for further participation "Would you like to add on?"
- Using wait time "Take you time....we'll wait..." <u>AND</u> After a student responds, so others can process the idea. "Think about what she said."

What counts as argumentation?

*** Recall that argumentation can be thought of broadly – more than constructing a full argument autonomously—and also includes a host of other practices such as:

- making conjectures, finding patterns, making inferences (reason inductively)
- analyzing a situation; analyzing relationships
- breaking into cases
- using a counterexample
- testing plausibility of a conjecture or idea (testing with examples)
- distinguishing correct logic from logic or reasoning with flaws
- stating assumptions; stating definitions used; stating relevant previous results (and applying, as appropriate)

What can you talk about?¹

• Mathematical Concepts, Computational Procedures, Mathematical Reasoning, Mathematical Terminology, Symbols, and Definitions, Forms of Representation



¹ From presentation by Kelly Lenox

A generative list of questions and prompts to elicit arguments (reasons)

Sense making:

Why does it make sense that... ? Why doesn't it make sense that... ? What makes you think that he's correct? Are these the same? Are these different?

Analysis:

Can you show me where xx came from?

Can you show me where xx is in your diagram?

Can you see where John used the idea xxxx (e.g., Can you see where John used the idea that it was 3 times as large? Can you see how John represented the slope?) Are these the same? Are these different?

Direct prompts for an argument: How did you know ...? Why does that method work? Please explain your reasoning to us. Do you agree or disagree, and why?

Decision making:

Why did you decide to/ choose to ... ? Why do you think she...? (ask about the idea of another real or fictitious student) Could I/you have done xxxx instead? Why/why not? Am I allowed to xxxx? Why/why not? Why was that a good move/step?

What did s/he just show?

Defending, convincing, proving: How do you know your answer is right? Can you convince me... ? What convinces you that... ? Can you show me why that has to be true? John, did he convince you yet that... ? Can you explain why that must be so? Defend your answer. Prove it to me.

Sell me on that idea.

Other tips

- Start "easy access" or "low threat:" What do you think might happen? What's your guess? What are you noticing? - or - What do you notice in this picture?
- ** Have something visuals, diagrams, graphs, tables, equations, tiles, blocks -to talk about! It's easier to start explaining an idea when you have something to point to and can use words like "this" and "that part." ****
- Develop language that can be used to share ideas (model, practice)
- Be specific about what you want students to share, clarify, show or prove. Ask about *their* idea and *their* thinking. Ask something you honestly do not know.

Four Other Routines to Support Argumentation in Math Class²

1. Alike and Different

Prompt can be written at the beginning of class, or any time during class. Students make lists, or can use Venn diagrams to show differences and similarities. Follow up questions lead to "how do you know..."

Ex. A. How are these equations alike? How are they different?

 $x^{2} + 3x + 10 = 0$ $-x^{2} + 12 = 0$

Student may observe: "one opens up and one opens down." Or "one has no solutions the other has 2 solutions." You can follow up such statements with How do you know?

Ex. B. How are these two shapes alike? How are they different?

2. Noticing & Wondering

Prompt asks students what they notice about a diagram, scenario, equation, etc. and what they are wondering about. The "wonderings" turn into places for reasoning and argumentation. The "noticing" helps students make sense of what they are discussing.

• Say/write one thing you notice (or understand); one thing you're wondering about (give stems)

I'm noticing	I'm wondering

3. Mystery Number; Giving Clues

Version 1a: You give four clues for a number, equation, object, etc.

- 1. My number has 2 digits.
 - 2. The sum of the digits is 15.
 - 3. The product of the digits is 56.
- 4. The number is even.

² Several of the above are from McCoy, Barnett, & Combs (2013). *High-Yield Routines.* Reston, VA. NCTM. *Wondering and Noticing* is from Math Forum

Version 1b. *What's my shape?* (an elaborated version!) *Present one item at a time.*

What's my shape?

At what point do you definitively KNOW what the shape is?

- 1. It is a closed figure with four straight sides
- 2. It has two long sides and two short sides
- 3. It has a right angle
- 4. The two long sides are parallel
- 5. It has two right angles
- 6. The two long sides are not the same length
- 7. The two short sides are not the same length
- 8. The two short sides are not parallel
- 9. The two long sides make right angles with one of the short sides
- 10. It has only two right angles

Version 2: Students can create the clues and give to a partner to solve. Develops vocabulary; strengthens inference skills; allows you to discuss redundant information.

4. How do you know?

Give students a question that may or may not have an obvious answer, but that requires reasoning. You can do private think time, pairs-share, or sharing with the whole class. You can model writing a response; have students write a response. There are lots of ways to do it.

Ex. 1: What shape is this? How do you know? Or: Is this a rectangle? How do you know?



Ex 2: How many solutions does this system have? How do you know?

$$2x - 5y = 3$$

 $4x - 10y = 1$

Ex 3: How do you know that 2(x - 3) = 2x - 6?

Ex 4: Which expression represents a larger value? How do you know?

$$(x^2 + 3)$$
 - or - $x^2 + 6x + 3$

Is It A Half?¹ Task Overview and Task Cards

Purpose:

To introduce the concept of $\frac{1}{2}$. To have students extend intuitive notions of $\frac{1}{2}$ (split into two equal pieces) to thinking about $\frac{1}{2}$ in terms of the *area* of a shaded figure (even if one cannot see two equal pieces).

Materials:

Bags of cards to be sorted (1 bag per pair) Large paper, divided into two parts down the middle. Left side is labeled Half and right side is labeled Not A Half. (1 paper per pair) Optional: tape or glue

Description of Activity:

- Each pair of students is given a bag of cards shaded in different ways.
- They are also given a large piece of paper with a line down the middle. One side is labeled **Half**, and the other is labeled **Not A Half**.
- In pairs, students take one card from the stack, discuss whether it represents and half or not (and why) and then place the card on the Half or Not-a-Half side, as appropriate.

The next pages contain some shaded examples of these rectangles for sorting. The last page has a template so more can be created.

One Possible Implementation:

Launch: With whole group, discuss ½. Where in everyday life do we see one half? Brainstorm and record on poster paper. Introduce concept that we say something is one half if it is half of the area (or length or volume) of the object. Discuss one or more examples that students had generated in terms of the *area*.

Activity: Students work in pairs. (see above)

Discussion: Whole class reconvenes. There's a large post-it paper or other public display labeled Half and Not-a-Half. The teacher uses cards students have, or new cards from a bag, and asks different students to come up to (a) place the card and (b) explain why the card is or is not a half.

Listen for:

- How students are making arguments
 - Matching pieces to show equal areas, or show by matching that shaded and unshaded are unequal
 - Showing 8 of 16, or not 8 of 16
 - Relating one card to another card (e.g., This one is like this one, except...)
 - Listen for misconception that it *has* to be symmetric

¹ Task created by Sarah Edwards, 2015, Manchester Public Schools, CT.

Video Viewing Is It a Half?

You will have the opportunity to watch two segment of video from a lesson where the teacher is trying to develop students' understanding of the concept of ½. The students are in third-grade and are working with a teacher-researcher.

As you watch, please think about the following questions, and write some notes. You may also wish to write notes directly on transcript, which is also included as a handout.

1. What kinds of questions and prompts do you see being asked?

2. How is argumentation being supported?

ls it a Half? Video Transcript

Clip 1

Brief description of focus of video: This is a 2.5-minute clip. We see two girls working together on a sorting activity where they are to determine whether or not certain shaded rectangles represent ¹/₂ or not. A classroom researcher is working with them, asking questions. The clip starts near the beginning of their work together (after place 6 cards) and they have some misconceptions about what ¹/₂ means.

CLIP 1: Is it a half?

[The two students and researcher are looking at a paper split in half with one side saying "Half" and one side saying "Not a half." There are smaller papers with different numbers of boxes shaded in on each side of the line.]

00:05

G1: Nope. Nope. Not even. No.

00:05

- R: Do you guys want to...can you explain why when you say it's not a half?
- G1: If it's a half, it would be like equal parts of the one you just saw.
- G2: [pointing to one of the cards on the side labeled "Half"] 1, 2, ...
- G1: It has to be the same size as this one [pointing to sheet of paper under "Half" side]
- R: It has to be the same size as that one on each side?



G1 and G2: Yeah.

- R: So, this one's a little more obvious, right? Can you explain why that's not a half maybe you want to do?
- G1: That's not a half because it has to go all the way or else it's just covering the whole thing.
- G2: Well, could it be like, could it cover this side too...to be a half too?
- G1: Yeah.
- R: Okay, so you guys agree if it covered either of those ways it would be a half?
- G1: Because that's 6 and that's 6 [points to a paper under the "not a half" column]
- 00:49
- R: Oh that's interesting. Okay, so what about this one? [*points to the first card in the "not a half" column, recreated here*] If you don't mind me asking about this one here.

- G1: That's not it because these two have to be together.
- [G1 points to two columns of un-shaded blocks on the sheet of paper and motions they should be next to each other.]
- R: Okay, would you say...so, could you possibly say that because...
- G1: Because half has only 2 sides, that has 3 sides.
- 01:10
- R: Ohhh, that's a really interesting idea: half has to have 2 sides, but not 3 sides. That's really interesting. So in math..it's a little...I see what you're saying because things like half moon that you talked about, you can't have the middle slice. In math, you can have half of something if it just equals half of the area.
- 01:30
- G1: Yeah.
- G2: Ohhhh.

R: So, it doesn't matter. So, let's slow down just a little. So, it doesn't matter if it's exactly two sides, but if it's half of the area. So let's look at this one.

[R points to the paper on top of the "Not a half" column]

- R: And let's decide...we have to argue either it's not half of the area, or it is half of the area. Because just saying it's not split in two sides doesn't convince us yet. Okay? So...
- G1: [*pointing to the sheet of paper*] Well, it has to be like that. It has to be the same as these two but they're still not.

01:59

- R: Still not what? You're saying that they're...
- G1: Still not the half.
- R: Still not the half? Can you tell me about the area though? Because that's going to be the most important part.
- G1: Area, well, they do have the same area.
- R: What has the same area?
- G1: These white ones. [points to un-shaded regions on either side of shaded block in the center of paper]
- G2: They have to be...[inaudible]
- R: Okay, so you're saying that...so what has the same area?
- G2: Umm.. these two have the same area [*pointing to shaded blocks*] but these don't [*pointing to un-shaded blocks*]. So that's why it's not equal.

02:25

- G1: They have to be together [pointing at white, un-shaded regions].
- R: They have to be together. Okay, so we need to, I need to find a piece of paper or something.

Clip 2

What makes something 1/2? (Especially when it doesn't "look like it"?)

00:06 - 2:33

- R: [*drawing squares and splitting them into 4 smaller squares*] Okay, so I think you guys agree if I do this and shade over here [*shades 2 out of the 4 smaller squares in the square*] you are going to tell me half is shaded.
- G1 and G2: Yeah.
- R: Okay you agree to that. Now what if I go like this [*draws another square*] and I cut it into 4 again [*splits square into 4 smaller squares*] and I do this and this [*shades 2 smaller squares in the square that are not next to one another*]. Is half of the area of that square shaded?



G1: Well actually it is because that is the same size as that one. [*pointing to smaller squares* within square]

00:30

- R: [inaudible] piece together are the same size?
- G1: Well, no it's still not half. Because...
- G2: Because they have to be the same size, same shape,....
- R: Oh, but if I do my shaded pieces, there's two. You see those are the shaded pieces?

[draws the two shaded smaller squares on their own outside of the larger square]

- G1 and G2: Oh yeah.
- R: And the nonshaded is like this at least, right? [*draws nonshaded smaller squares on their own outside of the larger square*]
- G1: Oh yeah. So it is half.

- R: It is half in that case. [*G1 has picked up a card that had been in Not A Half*] Now, so are you now agreeing this one is half, too?
- G1: Yeah. She agrees [referring to G2].
- R: [asking G2] Okay so what do you think? You're not sure yet?
- G2: It looks like it.

R: It looks like it. So the way we decide half is going to be based on area.



- G2: Actually, it kind of makes sense because these two are together [*pointing to shaded region from previous example*] and these two are together [*pointing to shaded region from original example*]. These two may not be together [*pointing to nonshaded from original example*] but that still doesn't mean it has to be.
- G1: Yeah, but it would make more sense if they're together. But, it can still be like that.
- R: Okay, cool. Okay, well keep going. What's the next one you want to do?
- G1: That one's not equal. [pointing to new example]



- G2: Yeah, because it's all like over the place [*pointing to shaded squares that are not all touching*]. [G1: It's all over the place.] And, and there's one like one [inaudible] shaded and one [inaudible] shaded...
- G1: When you see it you should know what it was.
- 01:45
- R: So, just by looking at it, it doesn't look like the same amount of area is shaded as not shaded, but how could we know for sure? Because it's really hard to tell.
- G1: We could actually count. 1, 2, 3, 4, 5, 6 [*counting the shaded squares on the paper*] 1, 2, 3, 4, 5, 6, 7 [*counting the unshaded squares on the paper*]. You already know you're over so that means it is not going to be a half.

2:05

R: Oh, okay. Alright. Can we go back to this one if you don't mind, now that we're understanding something new about areas and what half means? [*points to previous example]*. What do you think this one is?

G2: I think that looks equal.

R: You think that looks equal?

G1: 1, 2, 3, 4, 5, 6, 7, 8 [counting shaded boxes on example]. 1, 2, 3, 4, 5, 6, 7, 8 [counting unshaded boxes on example]. It's equal.

G2: I think that looks equal.

02:20

R: You're saying it's equal now?

G1: It's because these two are the same as that [*pointing to nonshaded and shaded boxes*]. If you just put them in the right order then it would make sense.

R: Okay, go ahead Cat what were you going to say.

G2: It's because there's two here, two there and two here [*pointing to shaded boxes*] and that's probably why it's equal because there's like two in each column.

G1: I agree with you.

Two Classroom Dialogues : Excerpt 1

Excerpt 1: The Brownie Problem

Students in Ms. Carter's class were exploring the concept of equivalent fractions. The specific problem follows: *The problem:* I invited 8 people to a party (including me). My mother got home with 9 brownies. How much did each person get if everyone got a fair share?

Sarah:

The first four, we cut them in half. [Jasmine divides squares in half on an overhead transparency. See figure below.]

	Ms. Carter:	Now as you explain, could you explain why you did it in half?
	Sarah:	Because when you put it in half it becomes eight halves.
	Ms. Carter:	Eight halves. What does that mean if there are eight halves?
	Sarah:	Each person gets half
Ms. Carter:	Okay, that each person gets a half. [Jasm	nine labels halves 1-8 for each of the eight people.]
Sarah:	Then there were five boxes [brownies] le	eft. We put them in eighths.
Ms. Carter:	Okay, so they divided them into eighths.	Could you tell us why you chose eighths?
Sarah:	It's easiest. Because then everyone will g	et each person will get a half and [whispers to
Isomina:	Jasmine How many eighths?	
Ms. Carter:	I didn't know why you did it in eighths. T	That's the reason. I
	just wanted to know why you chose eight	ths.
Jasmine:	We did eighths because then if we did eig 1/8 out of each brownie.	ghths, each person would get each eighth, I mean
Ms. Carter:	Okay, 1/8 out of each brownie. Can you	
	just, you don't have to number, but just	
	show us what you mean by that? I heard	
	the words, but [Jasmine shades in 1/8 of each of the five brownies not divided	
	in half.]	
Jasmine:	Person one would get this [Points to	
	one eighth.]	
Ms. Carter:	Oh, out of each brownie.	
Sarah:	Out of each brownie, one person will get 1/8	
Ms. Carter:	1/8. Okay. So how much then did they	
	get if they got their fair share?	
Jasmine/Sarah:	They got a $1/2$ and $5/8$.	The dialogue continues
Ms. Carter:	Do you want to write that down at the	0
	top, so I can see what you did? [Jasmine writes $\frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	From Kazemi, E. (1998). Discourse that promotes
	writes $72 + 1/0 + 1/0 + 1/0 + 1/0 + 1/0$	conceptual understanding. Teaching Children

Mathematics, 4(7), 410-414.

the top of the overhead projector.

Two Classroom Dialogues : Excerpt 2

Excerpt 2: Fractions and Factors (from Truxaw, 2004)

Ms. Reardon is reviewing for a test with her seventh grade class.

Ms. Reardon:	We're asked to rewrite 12 twenty-firsts in simple form. What do they mean? Don't give me
C.	an answer yet. But what do they mean by rewriting in simple form?
Steven:	I urn it into the lowest fraction possible that equals the 12 twenty-firsts.
Ms. Reardon:	Right. So, what is really getting smaller, not the fraction, but the?
Class:	Number
Ms. Reardon:	you guys list the factors of 12 for me? [<i>T. writes on board as she speaks</i>]. Factors of 12. Give me one pair. Lucas.
Lucas:	1 and 12
Ms. Reardon:	1 and 12. And I like to list them as pairs. I find it easier, so I don't leave anything out.
	[Lists on board]
Sheila:	6 and 2
Ms. Reardon:	6 and 2 [Lists on board.]
Roberto:	3 and 4
Ms. Reardon:	[T. lists on board]. Any others? [pauses for 5 seconds].
Ms. Reardon:	Do you guys agree with this?
Class:	Yeah.
Ms. Reardon:	Any more?
Class:	No.
Ms. Reardon:	I'd like you to do the same thing for 21.
Student:	1 & 21 [almost inaudible]
Ms. Reardon:	Uu- uh [indicating for S to stop speaking] thank you. Hands Garth.
Garth:	3 and 7
Ms. Reardon:	Okay [writes on board]
Joseph:	Um, 1 and 21
Ms. Reardon:	1 and 21. Okay. Any others? [pauses]
The	verbal exchanges continue similarly, finding the common factors of 21. Then
Ms. Reardon:	Now I want to knowcommon factorshmmmwhat do I mean by common? Amanda?
Amanda:	You see them more than once.
Ms. Reardon	Yes. We have it once here and once here. I'm going to circle and then write it over here [as
	a separate list]. Somebody tell me one number that appears in both lists.
Taylor:	One.
Ms. Reardon:	Breanna?
Breanna:	Three
Ms. Reardon:	[pauses, circling the common factors] No more?
Class:	[No response.]
Ms. Reardon:	Good. Okay. Put the extra comma in, in here. Now, I want the greatestcommon factor
	<i>[writes on board]</i> Sometimes abbreviated GCF. Greatest common factor. Everybody!

The dialogue continues...

From Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education*, *39*, 489-525.

Additional Thoughts on Questioning in Math Class

Asking questions that motivate student reflective thinking is an art. If our lessons are to be effective, we need to develop this art. It takes practice. As with most arts, there is not a set of hard and fast rules that work in all situations all of the time, but here are some general effective ideas to keep in mind for most times.

1. A "Try-to" List:

- Try to use effective pauses and wait time.
- Try to avoid frequent questions which require only a yes/no answer or simple recall.
- Try to avoid answering your own questions.
- Try to follow up student responses with questions and phrases such as, "why?" or "tell me how you know" or "think about how you can put Jim's response into your own words."
- Try to avoid directing a question to a student mainly for disciplinary reasons.
- Try to follow up a student's response by fielding it to the class or to another student for a reaction.
- Try to avoid giveaway facial expressions to student responses.
- Try to make it easy for students to ask a question at any time.
- Try to ask the question before calling on a student to respond.
- Try not to call on a particular student immediately after asking a question.
- Try to ask questions that are open-ended.
- Try not to label the degree of difficulty of a question.
- Try to leave an occasional question unanswered at the end of the period.
- Try to replace or enhance "lectures" with a set of appropriate questions.
- Try to keep the students actively involved in the learning process.
- 2. Questions to seldom ask: [The point here is that even though you *do* want to know the answers to these questions, the way these questions are phrased probably won't get you very far in learning what you want to know.]
 - "How many of you understood that?"
 - "Everybody see that?"
 - "You want me to go over that again?"
 - "This is a right triangle, isn't it?"
 - "Do you have any questions?"

- 3. **Phrases That Encourage Participation:** It's useful to have a handful of effective ways to start your questions that will motivate all students to participate. Here are some to try. What others can you think of?
 - "Don't raise your hand--yet; just think about a possible answer. I will give you a minute . . . "
 - "Everyone—picture this figure in your mind. Is it possible to sketch a possible counterexample to this statement? . . . I will walk around to look at your work and select 3 students to share their results with the class."
 - "Find an example for this statement and write it down. In just a minute I will tell you possible ways to check your example to see if it indeed makes the statement true."
 - "Put the next step on your paper and write a reason to justify this step. Raise your hand when you are ready and I will be around to check in on you."
- 4. Phrases That May Fail to Motivate: There are some questions that you might want to avoid. Why? Because often you end up answering your own questions . . . and "permitting" students NOT to participate—that is, students are not required to take responsibility to develop a response depending how the question is phrased.
 - "Does someone know if . . . "
 - "Can anyone here give me an example of . . . "
 - "Who knows the difference between . . . "
 - "Someone tell me the definition of . . . "
 - "OK, who wants to tell me about . . . "
- 5. Questions That Need Enhancing: Some common types of questions need some special care if they are to be useful in the math classroom. Otherwise, these questions do not provide much information to check students' reasoning.
 - Yes-No questions
 - True-false questions
 - One-word-answer questions
 - Questions that fail to motivate

Seeking, Explaining, Relating, Predicting, and Describing

The NDT Resource Center recommends that you use the top categories of questions from their list more frequently than those at the bottom. Ask students to:

- · Seek out evidence. ("What made you say that?")
- Explain. ("What caused Nixon's impeachment?")
- Relate concepts, ideas, and opinions. ("Compare germ-eliminating antibiotics to natural alternatives.")
- Predict. ("What will happen to Ahab if he continues to obsess about killing the Moby-Dick?")
- · Describe. ("What happens when Max is sent to bed without supper?")

Edutopia – Johnson

According to Robert Marzano's book, *Classroom Instruction that Works*, 80 percent of what is considered instruction involves asking questions. It makes sense then, that if we want to improve our effectiveness at teaching, of course we would start by improving our questions. I have thought a lot about this topic and I would like to share three specific actions that we can take to improve our questions. To begin with, we need to get students talking rather than the teacher talking. Second, prepare the questions when you plan the lesson. And third, scaffold the questions.

Common Core State Standards

Mathematical Practices

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.