# Module 5: Feedback to Advance Student Argumentation

## Goals: Module 5

Participants will

* Develop a deeper understanding of argumentation and its potential in the math classroom.
* Further develop strategies to support students in generating, extending and sharing their arguments (and understanding) as a discussion unfolds.
* Provide feedback (feedforward) to support mathematical argumentation based on analyzing student verbal and written mathematical arguments, using the structure of an argument.

In addition, participants will reflect on their learning across the 5 modules. For those in the Workshop Format, time and structure is provided to plan for the upcoming year.

## Overarching Questions for 5-Module Sequence

* What is a mathematical argument? What “counts” as an argument?
* What is the purpose(s) of argumentation in mathematics? In the math classroom?
* How do we organize our classroom to support student participation in the practice of mathematical argumentation, and to support them in developing their proficiency with argumentation (both verbal/interactive and written forms)?
* What does student argumentation look like at different levels of proficiency?

## Background Information:

In Module 5 *Feedback to Advance Student Argumentation*, we focus on supporting students in developing their arguments and getting better at producing arguments. Building on the attention to discourse in Module 4, we first engage participants in a role play with a teacher and students, where the teacher helps a small group make sense of one another’s arguments and come to new understandings. This work extends Module 4 as it turns attention to facilitating a small group discussion where students have different ideas and approaches. We then turn to providing feedback on students written work, which includes analyzing student work and writing *learning promoting* comments. Throughout this module, you will see an emphasis on providing feed*forward* to students on their reasoning and contributions in an effort to promote their conceptual understanding and proficiency with argumentation.

## Materials:

Copies of Handouts

 This includes sets of handouts for the role play

Slides to project

## Workflow Table for Module 5

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| --- | --- | --- |
| Session activity and focus | Estimated Timing | Materials |
| Monthly (1.5 hrs) | Workshop(3.5 hrs) |
| **Opening Activities**: PLC format: Participants share their “Bridging to Practice” workWorkshop format: Community Building and/or Problem Solving | 10 mins | (as appropriate for workshop timing) |  |
| **Activity 5.1** **Role Play Activity** with the Number Trick TaskParticipants take on the roles of teacher and students and role play a small group discussion with teacher facilitation | 30 mins | 50 mins |  |
| **Activity 5.2 Analyzing Student Work and Providing Feedforward** with the Number Trick Task | 40 mins | 65 mins |  |
| **Activity 5.3 Bridging To Practice**Monthly PLC Format: n/aWorkshop Format: Participants develop action plans for the upcoming school year | n/a | 65 mins |  |
| **Activity 5.4 Module (and Series) Closure**Participants reflect on experiences across the 5-module series and consider how to share ideas more broadly and sustain the work throughout the year | 10 mins | 30 mins |  |

## Implementation Guide and Possibilities: Module 5

### Opening Activity

### Monthly PLC Format

In the monthly PLC, you might organize participants into pairs or groups of three to debrief their Bridging-to-Practice work from Module 4 related to classroom discourse. For example, if participants audio recorded and transcribed segments of classroom discussion, the transcriptions can be shared and reviewed with an eye to funneling and focusing patterns, opportunities to supporting meaning making and reasoning, or specific points where participants would like help thinking of other ways the dialogue might have played out. Note that these discussions will likely touch on important aspects of not only discourse, but also key mathematical ideas, students’ mathematics, and lesson structure.

### Workshop Format

In the workshop format, you might use this time to engage participants in discussion of the community agreements, doing mathematics, or addressing questions that have come up but not yet been discussed.

As always, choose a problem you think will work for your particular group. We chose the Salary Problem, in part because there are so many different ways to represent the core idea of percent (and percent increase and percent decrease) and we wanted to give this aspect of argumentation some attention. This problem provides an opportunity to reflect on how – regardless of approach – the math idea at the heart of the problem must be a part of the argument. For this problem, the argument must address, reveal or highlight the idea that the *whole* is different at time 1 versus time 2, and consequently, 10% does not represent a constant amount, but rather different amounts of money.

### Module Objectives

Prior to Activity 5.1, the session objectives should be introduced.

Participants will

* Develop a deeper understanding of argumentation and its potential in the math classroom.
* Further develop strategies to support students in generating, extending and sharing their arguments (and understanding) as a discussion unfolds.
* Provide feedback (feedforward) to support mathematical argumentation based on analyzing student verbal and written mathematical arguments, using the structure of an argument.

### Activity 5.1 Role Play Activity

Overview: The purpose of this activity is to provide an opportunity to think carefully about facilitating a small group discussion. As students bring different ideas to the table, how might a teacher support them in developing their ideas and making sense of one another’s ideas. It also provides the opportunity to think carefully about what a student “really knows” based on a written work sample as participants take on the role of a particular student based on a work sample.

This activity first engages participants in doing the Number Trick task, a “trick” that is designed to provoke discussion about the conceptual underpinnings of the distributive property. Following this, in small groups, participants take on the roles of Teacher, who aims to facilitate a discussion, and Students, who are given a work sample from a middle school student and respond as if they are that student.

The role play is followed by a debrief where the group reflects on the experience, analyzing how the teacher elicited their thinking, helped students make sense of each other’s ideas, and helped to start building consensus and new understandings within the group. The group also might analyze further the thinking behind the student work samples.

Set Up – Selecting Task and Work samples: We have included a packet of 10 student work samples – from different 7th and 8th grade classes – for you to select from. In our work, groups of 3 used work samples F & I and groups of 4 used work samples B, C, & G. Other combinations can also be used. For example, you may decide B, C & H work better for your participants as it reduces the “range” of tools the students in the class seem to have. Note that these samples were drawn from 12 different classes, so the task has a few variations on specific questions, but the main questions are the same. Students I & J were in classes that presumably had been taught the distributive property.

Any task, with student work samples, can be used for this activity that is appropriate for your participants. You might find it helpful to use some from the Argumentation Resource Packets or other sites online such as insidemathematics.org. In selecting student work samples, we chose samples where we thought students had different approaches or degrees of understanding, and we could envision that a teacher would have to support the students in making sense of one another’s approaches and extending their approaches.

5.1.1 Becoming familiar with the Number Trick task

The facilitator reads the Number Trick task, and, with time, modeling how a teacher might introduce the launch. Participants work on the task individually for 5 minutes. The purpose is to make sense of the task enough to take on the role of teacher or student so there is no sharing of responses after the individual work time unless the facilitator deems it necessary for this purpose.

5.1.2 Organizing Groups

Participants should be in groups of 3 or 4, with one Teacher role and the other participants in Student roles.. If you need to make larger groups, groups of 4 and 5 can be made and the additional person can serve as a keen observer, taking note of the teacher’s moves, how those supported the mathematics, and considering additional questions or ways the conversation could have unfolded. These roles can be assigned in many ways, including drawing cards, putting a selection of candy on the table and whoever picks a particular candy, or alphabetically by middle name, for example.

Each Teacher needs the work samples for all students in the group.

Each Students needs a copy of only his or her work sample.

5.1.3 Role Play: Facilitating Argumentation

For the first 5 minutes of this activity, the Teacher reviews copies of the work samples of his or her students. The teacher may also choose to interact with them individually to learn some about their thinking. The Students read through their arguments (as presented in the student work samples) and convert it to a new format on an argumentation organizer, which is to help be the basis of the discussion. If an additional person is in each group, or if there are ample facilitators, that additional person can confer with the Teacher to talk about what they see in the work samples and how the conversation might unfold. *Notes on the Student Work samples are included below.*

After 5 minutes, the Teacher starts the conversation. You may choose to give the Teacher a sample sentence starter such as, “I see we have some different approaches to the Number Trick task. We’re going to talk about our ideas and work to develop together a strong argument for whether or not this “trick” works for all numbers, and why.”

Teachers then guide the conversation for about 10 minutes. We have provided a graphic organizer that can be used to support the work, but the goal of the conversation should not be filling in the organizer. Groups may not get to that point.

The Teacher may choose to make the goal the development of one shared argument from the group, or to have each student revise and extend his or her argument to make it better. Participants will likely notice that improving the arguments in many cases requires developing stronger conceptual understanding of what is happening with the Number Trick task.

5.1.4 Role Play Debrief

As you begin the Debrief, remind participants of the purpose of the activity as well – to think about how teachers support student participation in argumentation in “real time” and how teachers can support the development of student arguments. The following two questions are on a slide and can guide the debrief.

1. What stood out to you from engaging in the Number Trick Role Play Activity?
2. Is there a difference between facilitating discussions in general, and facilitating discussions that engage/support students in argumentation? If so, what are some of those distinctions?

We suggest first having a small group debrief to allow the group an opportunity to process its experience. Encourage an open dialogue about how Teacher questions supported Students in sharing their ideas and making sense of others’ ideas; about points when questions did not provide those opportunities and suggestions for alternate questions. Encourage the Teacher to share his or her rationale for why certain questions were posed and points where they were unsure of how to proceed.

In the whole group debrief, consider having a “report out” with one idea from each group for question 1, and engaging in discussion for question 2. As different groups have different work samples, you may want to provide all participants with the set of work samples before this point. Before facilitating this conversation around question 2, think about the group you are working with and how they might respond. We are hoping that both similarities and differences come out. A blank slide is included in the powerpoint slides so you can record similarities and differences (or future questions) as appropriate. A board or chart paper can also be used. Here is a short list of some similarities and differences. In the list, we treat differences as extensions of the similarities, which is just one of many ways participants might think about differences between the two.

|  |  |
| --- | --- |
| Similarities | Differences |
| Teacher must elicit student ideas and help students make sense of them. | When eliciting, there might be additional attention given to the student’s *reasoning* and making efforts to ensure his or her chain of reasoning is understood by others. |
| Students share ideas | Students share ideas but audience may be different. With argumentation, other students are an important part of this audience. (In some classes, the teacher is the audience.)  |
| It must be “ok to make mistakes” for students to share | It’s “OK to make mistakes,” *and* the teacher and class use these as opportunities to help everyone learn. The norm “it’s OK to make mistakes,” is modified -- “mistakes” are part of the process of argumentation where we’re working things out together. In fact, they’re not mistakes, but developing ideas.  |
| Students must listen to one another. | Listening to one another is more than just being quiet while someone else talks. Students must *listen to understand* another student, and think about the logic of their ideas. |

Participants may think that a discussion that involves argumentation must include written student work, or a graphic organizer such as the one provided for this activity. This is of course not the case. It may be helpful to note that students rarely can articulate in writing all that they are thinking, so the written record is a tool to help support a further conversation about ideas. Also, the act of recording and writing helps students reflect on their thinking. In this way, writing is also a process that supports the development of the argument, and not just records the final product.

### Activity 5.2 Analyzing and Providing Feedforward on Student Written Arguments

This two-part activity builds on Activity 5.2. Using student work samples on the Number Trick task, participants are asked to consider the set of student work as written work submitted. Perhaps the teacher is part way through the lesson and collecting work to plan for the next day. Or perhaps the lesson concluded and the teacher is looking to see what students seemed to take away (or at least what they could articulate in writing about what they took away). Part 1 of the activity asks participants to analyze the student work. Part 2 of the activity asks participants to provide feed*forward* on the task using the Stars and Stairs framework.

If you are working in the PLC format, you may wish to combined these parts for time purposes, and focus less on the component of argument to emphasize the newer ideas around productive feedback. We have included some draft slides (hidden) at the end of the powerpoint slides to help support that version of the implementation.

We recommend providing a handout with all student work samples used in the Role Play Activity, or providing participants with a fuller set of 9 work samples. We have included a pdf file of the set of 5 noted above (samples B, C, F, G, & I), as well as a pdf file of the full set of 9. We do not recommend having participants analyze 9 work samples – you should choose a subset, or ask them to analyze a certain number of the samples – but they may find value in looking across different responses and/or having some choice in which ones they analyze.

Activity 5.3.1 Analyzing

Participants are asked to look at each of the following work samples by responding to three prompts.

* **Identify the argument**
	+ What is the claim?
	+ What’s the evidence the student offers?
	+ What’s the warrant(s) that links the evidence to the claim?
* **Critique the argument**
	+ What are strengths of what the student did?
	+ Is the approach (chain of reasoning) mathematically sound? Are there logical gaps? Must the reader fill in connections or pieces of evidence?
* **Consider conceptual understanding**
	+ What can you infer about the student’s (developing) understanding of the distributive property?

The first two questions are designed to help participants *analyze* the student work as a mathematical argument. Note that this pulls the teacher away from “grading” the work or consider whether the response was sufficient as a response to the task prompt. The focus is what the student did with respect to *producing a viable argument.* It also focuses the analyzer on *what is there* and how it might be improved.

The third prompt is designed to help the participant think about how engaging students in argumentation is connected with developing their conceptual understanding of core concepts (here the distributive property). Argumentation does not have to be revealing of students’ conceptual understanding, but often in the classroom, teachers use argumentation for this purpose. These two purposes, *establish a claim is true* and *make sense of why something must be true*, go hand in hand.

Note that slides 20-22 of the powerpoint offer an example in relation to Student B. We suggest that an example be done together as a group before individuals, pairs or small groups work on the set of samples more independently.

Participants may generate some of the same comments or questions about these student work samples as they did in Module 1 when analyzing work on the Sums of Consecutive Numbers task. In addition, a key question that comes up is: **Does using the distributive property count as providing an argument?** This is a great question to raise. Our response is: for some classes, yes, to a large degree. In some classes for which the distributive property is known and understood, students can model both processes algebraically and use the distributive property to show that the two expressions are equivalent (produce the same output). Here, students must argue that their models for each process are correct (reflect the problem situation) and then that they have applied the right tool (the distributive property) in the right way (did the math right). If, however, the distributive property is not a known tool, then students cannot claim that the two expressions 4(n+2) (or (n+2)\*4) and 4n+8 are equivalent using this tool. They must argue that in a different way (assuming they are trying to take the approach of representing each process algebraically and showing equivalence).

The table below offers some commentary regarding the three prompts for this section. It also includes in the right hand column ideas for *Stars and Stairs feedfoward* which is an element of the next part of the activity.

There are a variety of ways to organize a discussion about these three prompts. To set up the Stars and Stairs component next, it is important to guide the discussion to analyzing the student work for strengths and areas of improvement. You might take “hot topics” from each group, record them on the board, and use these as guidance for some additional discussion; you might select one or two work samples that you’ve heard participants discuss in different ways to highlight. Keep in mind that this is setting up the next discussion, so keep participants in a developmental mindset – not about judging in this moment but thinking about how these students can be supported and continue to grow, whether in relation to their understanding of the distributive property or in relation to communicating their arguments.

Activity 5.3.2 Feedforward (based on the analysis)

This part of the activity begins with a brief overview of providing students with feed*forward* using the Stars and Stairs framework. The purpose of this segment is to provide participants with the opportunity to construct comments for students that fits within this framework.

Background: Though not discussed in the slides, the general approach for offering Stars and Stairs is based in the literature on formative assessment. Black & Wiliam (1998) have an excellent review that is well cited and still fully relevant. (See reference and link below.) The underlying idea is that for any child to improve he or she must (a) know what competent performance looks like; (b) know where they are now; and (c) have a sense of how to move forward. This leads to the Stars and Stairs framework where Stars help students understand what aspects of their performance are strong and the Stairs where students are provided with a sense of how they can improve. Teachers should always have in mind that students need to be able to act on that feedback to improve; if they can’t independently, teachers need to organize targeted learning opportunities. There are many other resources on formative assessment and providing good feedback that you might share. Participants may have also heard Stars and Stairs framed as a Medal and a Mission, or other phrases.

Please note: the purpose of Stars is not to “couch the negative feedback.” It is not about “sandwiching” the criticism between praise. That’s an evaluative stance. This is about *improving* and *providing feedforward.* For students to continue to improve, they need to know what about their performance is of good quality or worth keeping. Comments that are Stars reinforce the productive aspects of a performance and help a student see how his or her performance is aligned with expectations. Students cannot always do that on their own.

The introduction of Stars and Stairs can be followed by a brief check to reinforce key elements of the feedforward, namely that it is specific to the performance. Slide 26 provides that opportunity.

We have also included an example for Student B. You may or may not wish to use this, or you may ask everyone to do B first and have the example ready in case you want to offer it for some discussion. Participants could also dig into the activity at this point in pairs or small groups. We have recommended that they select 2 work samples and write Stars and Stairs for those samples. Encourage participants to draw on their analyses from part 1 as they write the Stars and Stairs.

Variation: A variation on this activity is to ask specific groups to write Stars and Stairs for a particular component of argumentation – the claim, the evidence or the warrant. In this way, participants think carefully about what the student is doing well, for example, with respect to the claim, the evidence or the warrant.

Table of Work Samples and Comments about the Arguments, Understanding, and Stars & Stairs

|  |  |  |
| --- | --- | --- |
| Sample | Argument | Potential Star and Stair comments and what might be of interest to discuss, including |
|  | *(there is no A)* |  |
| B | Claim: Jessie’s two answers always equalArgument: Evidence: none offered (May themselves have used the fact that you indeed do add more in the second trick.)Warrant: when you add after multiplying, instead of multiplying and then adding, you must add more [to have the result be the same]*A general appeal to knowledge about operations.*  | This student is using general reasoning that you must add more the second time because you added *after* multiplying*Star: You are doing a nice job thinking about the process and how that process would work regardless of the number you put in.**Stair: You’re saying you “add more.” Can you think about how much more you have to add for the second process and why?* *And what if you change the rules from doubling to tripling? You still add more, but how much more will change.* *How do we know how much more to add?*  |
| C | Claim: claim not stated; implicitly student indicates “yes”Argument: regardless of the rules (first or second process) you end up adding 4\*2 each timeEvidence: You need to add 8. 8 is 4x2, and in the first equation, you end up with 4 x 2, it’s just that the 4 is smushed in with the 5 when it gets doubled  | *Star: I can see how you’re thinking very carefully about how 8 gets added with each set of steps. Excellent thinking.* *Stair:**You’ve argued you really add 8 each time. I’m wondering about the doubling part. The original number is doubled in the second process. Is ti doubled in the first process?* *You’re doing a good job describing what is happening. Can you try to show (with a picture or diagram) how the two rules work? That might help you better communicate your ideas.* *Make sure your reader knows whether you think the trick will always work or not.*Of interest – the ways of expressing ideas – using “smushed in”. Example of students using the language they can to share their ideas.  |
|  | *Suggestion: discuss D & E as a pair* *Same “answers”**VERY different reasoning/support*  |  |
| D | #1 *asking about 1 – 10* Claim: YesEvidence: I tried them all in my head [and they ended up being the same]Warrant: (implicit) exhausted all cases: if I do all the relevant cases, and it’s true for each one, then it’s true for the claim.Note that this is not particularly sophisticated, but is a valid way to show this claim is true. A#2 *asking about all numbers* Claim: YesEvidence: I tried positive and negative numbers and they worked (also shows work for two examples)Warrant: (Implicit) if you try different types of numbers, and an example of each type works, then you know it works for all numbers (or perhaps at least for all numbers of that “type”) Explicit: no one has found one that doesn’t work, so it must always work Note that this is not a valid approach to demonstrating this claim is true. No matter how many different numbers, or different types of numbers you try, you cannot infer it is true for all numbers.  | Of interest - Student tried positives and negatives. It is productive that the student realizes that “just examples” doesn’t work. They are thinking generally in some sense by thinking about types of numbers.The student is, at least to a small degree, acknowledging that not all numbers were tested, and not asserting that it has been proven true for all numbers. The student notes that “the preponderance of evidence” is for the claim – “no one has found a number that didn’t work” – and fact that multiple methods were used. This preponderance of evidence is more similar to how we establish something as true (or a working truth) in science. *Star: I like that you were thinking about the fact that we have to show this is true beyond just 1 to 10. I see you testing positives and negatives and looking at your classmates’ values and approaches. This is a good way to personally convince ourselves.* *Stair: The testing – which could go on endlessly – helps us think it’s true, but doesn’t show it must be true for all numbers. Can you think about what’s going on with the math so that the two results are always the same when you use the same input? What are you doing to the numbers so that it works out each time?*  |
| E | #1 (Claim of 1 – 10)Claim: YesWarrant: “because it's the same exact thing”. The student doesn’t show that the two processes are the same, but uses that as the reason the outcomes are the same. (Two processes that are the same will produce the same result for the same input)Evidence: The student point out that “add four and double” from the first process is the same as adding the 8, which is a double 4, in the second process. So the evidence is a doubled 4 is 8. #2 Claim: YesWarrant: because my argument in #1 had nothing to do with the fact that it was 1 – 10 Evidence: shows example with 11 and gets the same answerWarrant 2 – because A\*2 + B\*2 is the same as (A+B)2*So the student is now representing each process and continuing to argument they are the same.*  Evidence – trying to show both equal F, but very muddled  | Verbal explanation of the 8 as well as saying you double in both cases. (doubled 4 is 8)Second part – is reflecting on initial argument and notes fact that 1-10 had “nothing to do with it.” Note that this student has really nice intuition! Recognizing that the particular input is irrelevant is insightfulThe symbolic work at the bottom is messy and doesn’t build to much, but shows this student has a decent sense of how to represent processes symbolically. The student even perhaps generalizes beyond adding 4 to claim that (A+B)\*2 =F and (A\*2) + (B\*2) = F though above s/he says A is the “thought number” and B is “4” so s/he may not really be thinking generally.*Stars: In the second question, you did a nice job expressing the two processes as A\*2 + B\*2 and (A+B)\*2. And I like how you told us what you're a represented and B represented.* *Stairs: (assuming the class hasn't provide the distributive property yet)* *The key to your argument is showing that A\*2 + B\*2 and (A+B)\*2 are equal. We haven’t yet shown this in class. Can you think of how to convince someone that A\*2+B\*2 = (A+B)\*2? A drawing or other representation may help.* |
| F | Claim: YesWarrant 1: “because in my data, both equations get the same numbers whatever n is”Evidence: tested n = 3 and 8 in the table, by following the steps of the rules, and seems to test 4, -3 and 120 in their calculations.(Has two equations in top left, but doesn’t really do anything with them. Doesn’t seem to be part of the argument at the top)Warrant 2: (bottom part of page) because the rules are the same (even though they look different). Here the student seems to be referencing the rules 2n+8 and 2(4+n). Evidence: drawing of 2n + 8 and then showing that n + 4 x2 would give the same -- two n bags and 8 unit squares. *This is left to the reader to “see” that these two processes are being modeled AND that these two processes produce the same amount*  | Pictorial representation of equivalence of expressionsOf interest – first part of claim rests on data; second part, by contrast, rests on having expressions and showing they are the same using a diagram A couple questions that could be rasied are; Does the student think s/he needs both? What is convincing to him/her?*Stars: It looks like you did a very nice job noticing what the process was for each rule of the trick and then figuring out how to write that as an expression. (You have 2(n+4) and 2n+8). Great mathematical thinking.* *Stairs: You give two different reasons why the trick works. You first say it’s because the data shows you always get the same number. You also say the rule are the same, and have a diagram that may show this. The data is helpful testing, but doesn’t show the trick always works. Try to make even strong your argument that 2n+8 is the same as 2(4+n). You’re almost there – really show the reader how you know these are the same. You can build on your diagram and make it clearer.*  |
| G | Claim: you get “the same result” so implicitly claiming yesWarrant: (though written differently) the equations always produce the same resultEvidence: their table of values (shows for 10 values), which generated their graph it seems. Note that they leave it to the reader to know that they are looking at 2 superimposed lines. The argument would be: if the lines are the same, the trick works for any value.Warrant #2: they ended up being the same because it was just a matter of parentheses*Not well stated, but the student seems to be getting at – once you “deal with” the parentheses, the expressions no longer look different?* | **NOTE:** May be worth noticing that that some students argue the OUTPUT is always the same, or is the same for many values, so much e always the same. This is not a valid approach. Other argue the PROCESSES produce the same output not matter the input. This is a valid approach. This student is looking at “same outputs.”Of interest – this is essentially the same as testing many points and assuming it’s true for all. Will the teachers judge this of higher, lower, or the same quality than one where students tested many values? Are we lured by the graph and equations?[[1]](#footnote-1)Note also that the student “reverses” the axes from our conventions, with x being on the vertical axes. As the equations are 1-to-1, there’s nothing wrong with this graph but because it goes against convention, a key/labeling would be helpful.*Stars: I can see you looked at these two processes in many different ways –patterns in a table of values, graphed to see a trend, and wrote each process as an equation (2x+8 and 2(x+4). This is all good mathematics.**Stairs: I’m wondering: how is it that these rather different looking equations end up giving you the same result? I see that it does give the same result for many, many numbers. But why does it? How come it works out each time?* *(could also ask: how do you know that the two equations you have represent the two different rules accurately? How did you decide how to write these?)* |
| H | Qu #2 Claim: Get same result for my test value (they match)Warrant: you’re doing the same thing in both tricks really – same processes should produce the same resultsQu #4 Claim (1 – 10): YesWarrant: implicit (not stated): if I test each number, and follow the two rules correctly, and get the same each time, then it works for 1 - 10Evidence only: tested all (though actually skipped 6) and underlines results to show they are the same#5 Claim (any number): YesWarrant: “both are the same” (Or more fleshed out, the student seems to be basing the claim on the following: if I can transform the expression for rule 1 into another new expression, and then the expression for rule 2 into *the same new expression,* then the two rules are “the same” and the trick will always work.)Evidence: (N + 4) times 2 is A and you’ll get a 2N + 8out of that (read across) [the student shows the 2\*4 and an arrow from the 2 to the N and writes 2N)The second process is (N \* 2) + 8, and there is an arrow to 2N+8. *The student states draws an arrow – and leaves step of 2N = N\*2 to the reader.* *Handwaives Distributive Property* note: this class had not been taught yet the distributive property. It is not known whether the student’s connection of (N+4)\*2 to 2N+8 is really understood, or if the student is kind of doing a matching game of values/numbers.   | Language of “same thing in both”Claims (N\*2)+8 “is the same” as 2N+8 = A. As a teacher, one might want to follow up with this student on what “the same” means to him/her in these different context.Of interest - This class hadn’t “done” the distributive property *Stars:**You clearly have understood the rules and applied them accurately to many examples. You’re also noticing some similarities in the two rules.**Stairs:**There are two important areas to work on:**1) Can you explain more about how these rules – that are different – end up making “the same thing” or being “the same”? What’s different about them? How does that produce the SAME VALUES?**2) I’m wondering where your equations in #5 came from. Can you explain how they represent the two rules?* |
|  | *These next 2 samples were from a class that “knew” the distributive property.*  |  |
| I | Claim #1: Yes the equations are the sameEvidence: manipulation of the equations Warrants: nods to idea that these manipuliatsion were made possibly by substitution and commutative property. These warrants, however, are not connected with specific manipulations.Claim #2: Yes the equations are equal (and so it works for all values)Warrant: (this is what we think the child’s implicit argument is) I get 0 = 0 when I set them equal to each other and manipulate, so that must mean they are the sameEvidence: showing manipulations to produce 0 = 0 Note the student’s work begins by substituting in x = 40. What the student has really shown is that 2(40+4) equals 2(40)+8. So this verifies the two rules produce the same result for x =40. | Student uses, but does not name, the distributive property. Names other properties.Of interest – does the student think s/he needs to include the example with 40 to justify the result? What’s the role of that example?Does this count as a justification *without* naming the distributive property?*Star:* *I like that you represented the two processes in a general way using algebra.* *Stair:* *Can you explain more about how you knew that (x+4)\*2 = 2\*4 + 2x. What allows us to re-write the expression in this new form? Or can you find a way to show that you can re-write the left expression in this new form?*  |
| J | Claim #1: YesWarrant: when you write these “operations” algebraically (the student seems to mean rules), they are “the same” by the distributive property [note that this is the only work sample to reference this property as a warrant]Evidence: you can re-write the equation (x + 4)\*2 as 2x + 8. The second rule is 2x+8. These are the same. Claim #2: Yes my explanation does show it for any number **note**: this is the second student to read the prompt as a critique of his/her above argument and to evaluate its applicability to other numbers.Warrant: Doesn’t give a warrant for why the explanation above holds, but does restate the warrant that the two e expressions known to be equivalent because the expressions are identical after manipulating one using the distributive property.Also left unsupported is how the student knows s/he has written the correct expressions for the process.Evidence: Has expressions for both rules and an equation tucked in the middle where s/he shows arrows that make us think of the distributive property. Doesn’t give evidence for non-stated warrant, but chooses to add some examples.Note clear is the student thinks the four examples are part of the evidence. | Student basically only names the distributive property – entire justification rests on that. Teachers will have to decide if that’s sufficient for their class. This likely would be for mathematicians in relation to this problem, along with showing the algebraic expression for each rule. (The student does have some calculations for fractions, imaginaries and the like, but these do not contribute to the argument.)Of interest – how will teachers think about this? What does this student think a justification is?Important note: What count as Stars and Stairs here depend on what can be assumed prior knowledge and skills. It’s not clear, for instances, whether it is pedagogically useful to ask this student to explain how the expressions represent the rules/processes. Students may take this as “obvious” at this point.*Stars: You did a nice job mathematizing each set of steps for the trick and then explaining that the distributive property helps you know that (x+4)\*2 is the same as 2x+8 (and therefore the trick works for all numbers – not matter what the input, you get the same output).* *Stairs: As a finer point on the language you use, notice that 2x+8 and (x+4)\*2 are not the “same expressions” (which you say at one point). They are equivalent expressions, and you have shown this by the distributive property. The expressions themselves are different. As you write your arguments, re-read to see if your language use is helping you convey exactly what you mean.* |

### Activity 5.3 Bridging to Practice

Monthy PLC Format: This is not applicable for this session, as it is the last module. However, as part of your Session Closure, you may choose a reflection or activity that helps participants think about these ideas in relation to their future work. We have included suggestions under 5.4 Session Closure.

Workshop Format:

One possibility for the Bridging to Practice activity is to have participants make a plan for the upcoming school year (or if the school year is in process, for what remains of the year). We suggest a two-part activity that begins with a reflection. Another possibility is doing a Photo-Album resource identification and organization activity. We describe that below.

**Action Plan Development Activity**

Reflection: The activity can begin with the following three reflection questions. (And handout is provided in the materials):

1. What will count in your classroom for a valid argument? (What qualities or criteria are important to you?)
2. How will these criteria be communicated to students? (What vocabulary and meaning making must be built up around argumentation?)
3. What do you expect at the beginning of the year? Where will growth be? (It may help to think of a question that involves reasoning/argumentation that you might pose to students. Consider how the students might respond at the beginning of the year? What growth might you see over the year?)

These questions, followed by discussion, can also provide a useful discussion about what other ways can you support students in engaging in argumentation and getting better at argumentation. Some ideas that might come up include: presenting (composite) student work samples for students to analyze; asking students to look at three work samples and discuss strengths and weaknesses of each, or compare across; explicitly introducing vocabulary related to argumentation, perhaps distinguishing between an explanation and argument, conjecture and theorem; use sentence starters and prompts for student-to-student exchanges; have some students – or teachers – model a discussion (that involves argumentation and focuses on reasoning); emphasize the idea of revising and revisions; using self evaluation and/or peer feedback; model with examples and non-examples; help students learn to *write* what they *say* and are thinking. How teachers will plan lessons might come up as well.

This also sets the stage for the second part of the activity where participants map out a plan for themselves. The Action Plan development activity builds on the work they may have done at the end module 4 as well that focused on routines. A handout is provided to help guide that work.

A range of formats can be useful during the development of Action Plans. Participants can work individual, in small groups, or develop work individuals and then share and get feedback from others.

The **Photo Album activity** provides participants with time to look back through the resources from the modules, as well as online at the Bridging Math Practices site, and identify and organize those resources for future use. If you are doing the Photo Album and Action Plan activities, you may choose to combine them in some way, or do the Photo Album activity first, as that activity can help participants brainstorm and review a wide range of resources and ideas before developing their action plans. Suggested timings are provided on the handout, but these can be easily modified. We included what we suspect is close to the minimum amount of time required to make it a valuable activity.

### Activity 5.4 Session Closure

Depending on your format and time, this closure activity can comprise various activities. We are including suggestions for a “Photo Album” resource-solidification activity, a Concept Map Activity and/or Individual Reflection.

In the individual Reflection, we have emphasized helping participants think about what ideas they’ve learned that are important and that they’d like to share with others.

Another option to support a “looking back” and synthesis of the 5-module sequence is a Concept Map activity. Concept Maps can be powerful tools for providing means to (re)organize one’s ideas and to see previously unnoticed connections. They help to show the interrelationship among ideas. This activity can be done in a variety of ways. The materials include a set of “cards” that you will need to print and cut out that will be the “concepts” used in the map. Alternate, you could have participants generate their own, or you could use a subset of those. This activity will likely work best in small groups. Participants also will need a place to create the map. They can post to the wall, create on large post-it paper, use poster board, etc. You will also need glue, tape and possibly scissors.

# References: Module 5

Paul Black and Dylan Wiliam, “Inside the Black Box: Raising Standards Through Classroom Assessment,” Phi Delta Kappan Vol. 80 (2), October 1998, pp.139-148. Available at

<https://weaeducation.typepad.co.uk/files/blackbox-1.pdf> or https://www.michigan.gov/documents/mde/Inside\_the\_Black\_Box\_184495\_7.pdf

# Additional Resources: Module 5

1. Some might argue that once you have the data points, you see the line, and so you know it’s true for all values. The question is *how do we know that this pattern is linear.* Indeed, *if* it is linear, *and* we have 2 points, we can then extend the line (here, lines) and effectively know/test all values. However, in this case, we do not know the process is linear. An argument must be made that it is linear in order for that approach to produce a valid argument. [↑](#footnote-ref-1)